Period Doubling Route to Chaos in SiGe IMPATT Diodes

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***Abstract*—An instantaneous electric model for a SiGe IMPATT diode is calculated here from the physical quantities provided by the foundry. The model is used for the nonlinear simulation of IMPATT-based circuits, which are prone to instability. In this letter, a bifurcation analysis is carried out in order to determine their dynamical response, as a function of any suitable parameter. The technique has been applied to the equivalent circuit of the reflection measurement system, from which the IMPATT immitance variation curves are usually determined. The bifurcation analysis allowed the detection of a period doubling route to chaos, in good agreement with the experimental observations.**

***Index Terms*— Bifurcation, IMPATT diode, period doubling, chaos.**

I. Introduction

It is well known that IMPATT diodes present at microwave frequencies a dynamical negative resistance, based on transit time effects, and are mainly used in the design of oscillators and amplifiers at millimetric frequencies, when high output power is required. This design is usually carried out from immittance variation curves, as a function of radio frequency (RF) amplitude and frequency, normally obtained from reflection measurements at a vectorial network analyzer. This characterization of the device dramatically limits the information about its nonlinear response. In fact, IMPATT-based circuits are prone to instability, as has often been reported in the literature [1], [2], with the common observation of phenomena such as the onset of subharmonic and low-frequency oscillations. Chaotic responses [3] are also usually found in the experiment.

In this letter, physical parameters have been used to obtain an instantaneous model for a SiGe IMPATT diode, based on the one proposed by Gannet and Chua [4], and are well-suited for the nonlinear design of IMPATT-based circuits. The model is mainly a circuit implementation of the diode transport equations, with each element in direct correspondence with a physical mechanism. Here a bifurcation analysis technique is applied to a SiGe IMPATT-based circuit, justified by the common observations of frequency division and appearance of spurious frequencies, which are examples of bifurcation phenomena.

The period doubling route is one of the possible bifurcation sequences, leading to a chaotic behavior, when a given parameter μ is varied. Successive period doublings take place, with decreasing parameter ranges (μ_n, μ_{n+1}) of 2^nT period until, at a certain parameter value μ_∞, the period doubles ad infinitum, obtaining a chaotic behavior. Feigenbaum [5] has shown that the parameter series $\{\mu_n\}$ converges fast to μ_∞. In order to analyze this convergence, he defined the ratio

$$\delta_n = (\mu_{n+1} - \mu_n)/(\mu_{n+2} - \mu_{n+1}) \tag{1}$$

with δ_n agreeing with the value $\delta = 4.669\,201\,6$ after just a few period doublings.

This period-doubling route to chaos was obtained here when modifying the input generator amplitude of the classical measurement system of the diode immittance. Due to the simplicity of the corresponding equivalent circuit, a time-domain analysis could be carried out, applying the technique of the Poincare mapping [5] to trace the system bifurcation diagram. The Feigenbaum ratio was evaluated, obtaining a good agreement with its theoretical value. The results also confirmed former experimental observations.

II. Impatt Instantaneous Model

In the following, an instantaneous model for a SiGe IMPATT diode will be derived, under the assumptions of p-i-n doping profile, low current bias, and negligible diffusion effects. The model is composed of two equivalent circuits [4] (one for the avalanche region and the other for the drift one) in a series connection (Fig. 1), and it accounts for the diode current and voltage variations over the static operation point, given by the bias current I_0.

Two nonlinear elements are considered: the avalanche inductance L_a and the quadratic *voltage-controlled* voltage source V_N. In the drift region, the *current-driven* current source I_e accounts for the drift delay and, for negligible diffusion effects, can be calculated as a short time average of the avalanche current (Fig. 1). The two capacitances C_a and C_d, respectively, account for the avalanche and drift region displacement currents, and their calculation from the diode cross-sectional area A and the respective widths l_a and l_d is straightforward (Fig. 1).

The two constant factors Fac_1 and Fac_2 are given by

$$Fac_1 = \frac{l_a v_s}{\alpha'(E_c)} \tag{2}$$

$$Fac_2 = \frac{\alpha''(E_c)}{2 l_a \alpha'(E_c)} \tag{3}$$

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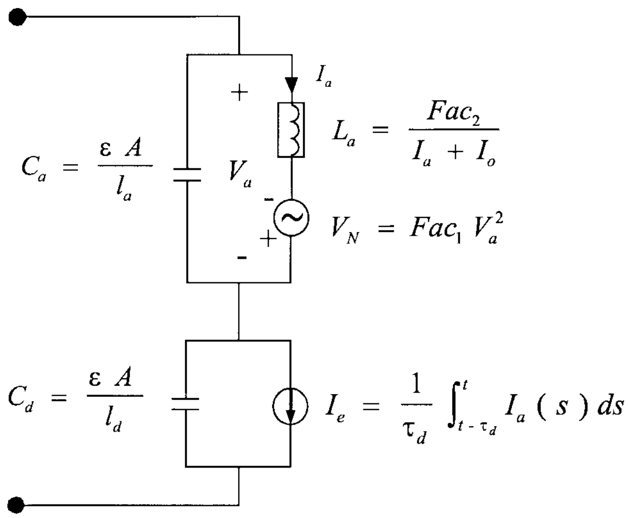


Fig. 1. IMPATT diode instantaneous model. The delay τ_d is obtained from $\tau_d = l_d/v_s$.

with v_s being the saturated carrier velocity and $\alpha'(E_c)$ and $\alpha''(E_c)$ the first- and second-order electric field derivatives of the effective ionization rate, evaluated at the static field E_c. The latter is calculated from the infinite multiplication condition [4]. The electron and hole ionization rates α_n and α_p are related to the electric field through the semiempirical functions [6]

$$\alpha_{n,p}(E) = \frac{eE}{Wi_{n,p}} \exp\left(-\frac{Fi_{n,p}Fr_{n,p}}{E^2}\right) \tag{4}$$

with e being the electron charge, $Wi_{n,p}$ the high-field ionization threshold energies, $Fi_{n,p}$ the threshold fields compensating for Coulomb scattering, and $Fr_{n,p}$ for optical-phonon scattering. The resulting values were $Fac_1 = 1.30\ 10^{-12}$ V $\cdot$ s and $Fac_2 = 2.54\ 10^{-3}$ V^{-1}.

In this letter, a small-signal equivalent of the model in Fig. 1, to be used in preliminary designs, has also been derived. The quadratic voltage source disappears under small-signal conditions and the nonlinear inductance takes the constant value $L_a = 1/I_0$. In the drift region, the gain g_e, corresponding to I_e, is calculated by introducing a sinusoidal waveform in the time averaging

$$g_e = \frac{2}{\omega\tau_d} \sin\omega\tau_d \left[\cos\frac{\omega\tau_d}{2} - j\sin\frac{\omega\tau_d}{2}\right]. \tag{5}$$

Then the calculation of the diode input impedance provides the following expression:

$$Z(\omega) = -\frac{(1-\cos\omega\tau_d)}{(L_aC_a\omega^2 - 1)C_d\tau_d\omega^2} - j\frac{(L_aC_T\omega^2 - 1)\omega\tau_d + \sin\omega\tau_d}{(L_aC_a\omega^2 - 1)C_d\tau_d\omega^2} \tag{6}$$

with $C_T = C_a + C_d$

From the inspection of (6), the lower frequency limit for negative resistance is determined by the resonant frequency of the avalanche circuit, while the upper one is given by the inverse of the drift delay. On the other hand, the imaginary part is always capacitive above the avalanche resonant frequency, which is in good agreement with the results of [6].

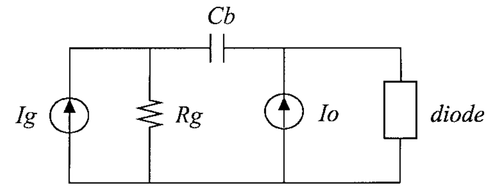


Fig. 2. Equivalent circuit of the reflection measurement system. $R_g = 50$ Ω. C_b, bias block.

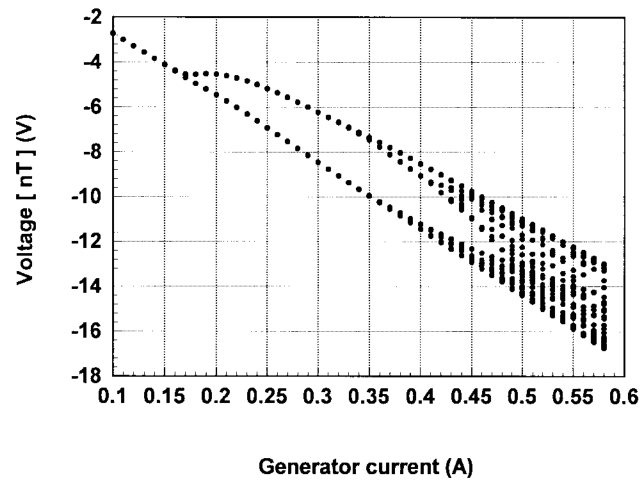


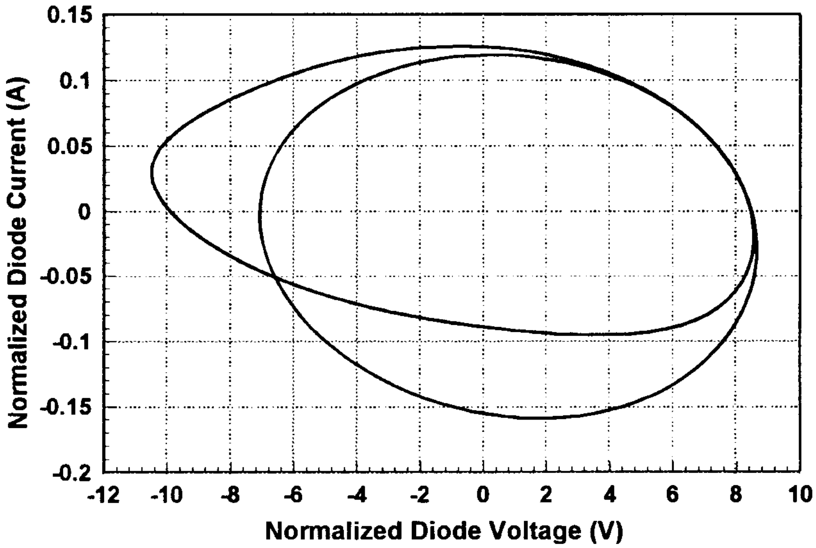
Fig. 3. Bifurcation diagram, through Poincare mapping.

III. Analysis Method

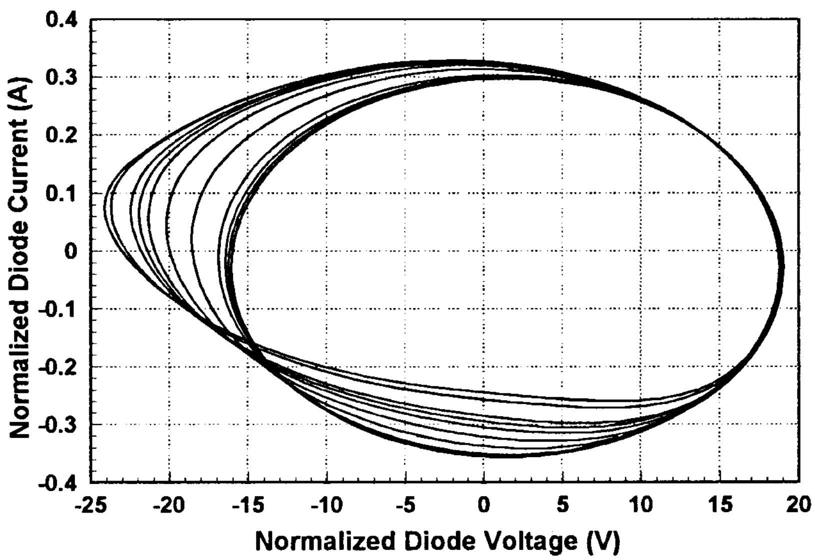
The technique of the Poincare mapping [5] has been used to obtain the bifurcation diagram of the circuit in Fig. 2, as a function of the RF amplitude. This technique is based on the evaluation, for every parameter step, of the circuit steady response for a number of integer multiples of the input generator period. Clearly, for periodic solutions of the same period T as the input generator, this analysis will provide a single point, while for periodic solutions of period nT it will provide n different points.

For 50-Ghz input frequency, in the negative resistance band, the resulting bifurcation diagram is depicted in Fig. 3. It has been obtained by evaluating the diode voltage as a function of the current source amplitude, and it clearly shows, as the input power increases, a period-doubling route to chaotic behavior, which confirms the inaccuracy of the classical diode characterization. Chaos can be recognized from the characteristic *cloud* of points in the Poincare diagram, due to the loss of periodicity of the circuit response. The two first iterations of the Feigenbaum constant were calculated, obtaining $\delta_1 = 2.57, \delta_2 = 4.68$, which constitutes a good approach to the theoretical value.

In Fig. 4, the diode current to voltage variations, over the static operation point, have been represented for two different input generator amplitudes. For $I_g = 0.2$ A, the double



(a)



(b)

Fig. 4. Current to voltage variations, over the static operating point. (a) $I_g = 0.2$ A. (b) $I_g = 0.5$ A.

cycle [Fig. 4(a)] is in good correspondence with the frequency division by two. For $I_g = 0.5$ A, a chaotic attractor is observed [Fig. 4(b)], with its characteristic fine structure [7].

IV. Conclusions

In the present letter an instantaneous model has been derived for a SiGe IMPATT diode, from physical parameters. The model allows the nonlinear dynamical simulation of IMPATT-based circuits, and it has been used here in combination with a bifurcation analysis technique to predict their possible anomalous behavior. In this way, a period doubling route to chaos has been obtained when analyzing the equivalent circuit of a typical measurement system, used to determine the diode immittance variations as a function of the input amplitude. The simulation results are in good agreement with formerly reported experimental observations.

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